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## 高レイノルズ数一様等方性乱流における秩序渦： ウェーブレットの視点から

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### 1 Introduction

Wavelet analysis allows a sparse representation of turbulence. The wavelet transform decomposes a turbulent flow field into space-scale contributions. The small scale contributions are significant only in active regions but not in weak regions. If we can neglect or model the non-significant contributions, we can reduce the number of the wavelet coefficients to track turbulence significantly. A wavelet-based method to extract coherent vortices from turbulent flows has been introduced[1, 2]. It splits the vorticity field into two sets, coherent and incoherent vorticity. The coherent vorticity exhibits similar statistical behavior as the total vorticity. The incoherent vorticity reconstructed from most of the weaker coefficients is an almost uncorrelated random background flow. A new turbulence model, called Coherent Vortex Simulation (CVS), has been proposed[3]. It is based on a deterministic computation of the coherent flow evolution by the use of an adaptive wavelet basis and modelling of the influence of the incoherent flow.

The aims of this work are the followings;

- (1) We examine the Reynolds number dependence of contribution of coherent and incoherent vorticity to high Reynolds number homogeneous isotropic turbulence as *a priori* test on CVS.

- (2) We estimate how the number of the wavelet coefficients, corresponding to the coherent vortices, depends on the Reynolds number.

These are the key questions for the feasibility of the CVS approach. The details are found in ref. [5]

## 2 Wavelet analysis and coherent vortex extraction

First, we summarize the main ideas of wavelet analysis, the coherent vortex extraction and DNS datasets we used.

### 2.1 Vector valued orthogonal wavelet decomposition

Wavelets are functions well localized in both physical and spectral space. In particular, orthonormal wavelet analysis the fast wavelet transformation with linear complexity and has no redundancy. It unfolds a vector field into scale, positions and directions and decomposes it into an orthogonal wavelet series

$$\mathbf{v}(\mathbf{x}) = \sum_{\lambda \in \Lambda} \tilde{\mathbf{v}}_{\lambda} \psi_{\lambda}(\mathbf{x}). \quad (1)$$

From a vector field sampled on  $N$  equidistant grid points, we obtain the  $N$  wavelet coefficients by the fast wavelet transform. When scale becomes smaller ( $j$  increases), we have more wavelet coefficients.

### 2.2 Coherent Vortex Extraction

The wavelet-based Coherent Vortex Extraction (CVE) method[1, 2] is based on the following:

- (1) We consider the vorticity field rather than the velocity field, since it preserves Galilean invariance.
- (2) We consider the minimal but hopefully consensual statement about coherent structures: *‘coherent structures are not noise and correspond to what remain after denoising’*.

- (3) As the simplest guess, the noise is supposed to be additive, Gaussian and uncorrelated.

We briefly sketch the CVE procedure. Readers interested in the details may be referred to the original papers.[1, 4] An orthogonal wavelet decomposition is applied to the vorticity field  $\omega$ . A threshold based on denoising theory[6], which depends on the enstrophy and resolution of the field, splits the wavelet coefficients into two sets. The coherent vorticity  $\omega_C$  is reconstructed from few wavelet coefficients whose moduli are larger than the threshold. The incoherent vorticity which can be reconstructed from the many remaining weaker coefficients satisfies the equation  $\omega_I = \omega - \omega_C$  due to the orthogonality of the wavelet basis. In the CVE, we prefer the Coifman 12 wavelet, which is compactly supported, has four vanishing moments, and is quasi-symmetric.

### 3 DNS data sets at $R_\lambda = 167, 257, 471$ and $732$

We used the four DNS datasets of three-dimensional incompressible turbulence of  $k_{\max}\eta \simeq 1$  computed on the Earth Simulator[7, 8].  $k_{\max}$  is the maximum wavenumber of the retained modes,  $\eta$  is the Kolmogorov length scale.

The number of grid points and Taylor microscale Reynolds number for each DNS are listed in Table 1 of ref.[5].

## 4 Coherent vortex extraction for $R_\lambda = 732$

Now we apply the coherent vortex extraction method to the DNS data for the highest Reynolds number case.

### 4.1 Visualization

Figure 2(top) in ref.[5] shows the modulus of vorticity of the total flow, after zooming on a subcube to enhance structural details. Then we decompose the flow into the coherent and incoherent contributions. The coherent flow retains the vortex tubes present in the total vorticity, and well superimposes with the total one as shown in figs. 2(top) and (bottom left) in ref.[5]. In contrast, the incoherent vorticity is structureless (fig. 2(bottom right) in ref.[5]).

## 4.2 Velocity probability density functions

Figure 3(top) in ref.[5] shows the PDFs of the velocity components of the total, coherent and incoherent velocity. The Gaussian distribution, which is normalized so that it has zero mean and the same standard deviation as that of the incoherent velocity, is also plotted. The total and coherent velocity PDFs coincide well. We find that the incoherent velocity PDF is quasi-Gaussian with a strongly reduced variance compared to the total velocity PDF.

## 4.3 Vorticity probability density functions

The PDFs of the vorticity components are shown in fig. 3(bottom) in ref.[5]. The coherent vorticity PDF is in good agreement with the total one. They show a stretched exponential behavior which illustrates the intermittency due to the presence of coherent vortices. The PDF of the incoherent vorticity has an exponential shape with a reduced variance compared to that of the total vorticity.

## 4.4 Energy spectra

The energy spectra of the total, coherent and incoherent flows are illustrated in fig. 4 of ref.[5]. This shows that the energy spectrum of the coherent flow is identical to the total one all along the inertial range. In the dissipation range, we see the difference between the coherent energy spectrum and the total one, though the coherent vortices still keep a significant contribution for the range. For the incoherent flow, we observe that the scaling of the incoherent energy spectrum is close to  $k^2$ , which corresponds to an equipartition of incoherent energy between all wavenumbers.

## 4.5 Energy transfers and fluxes

Studying the energy transfer in Fourier space enables us to check the contributions of the coherent and incoherent flows to energy flux in spectral space. Using the decomposition of the total velocity  $\mathbf{v}$  into the coherent and incoherent velocity  $\mathbf{v}_c + \mathbf{v}_i$ , we obtain 8 energy transfer

functions and enegy fluxes for possible combinations between coherent and incoherent flows.

$$T_{\alpha\beta\gamma}(k) = - \sum_{k-1/2 \leq |\mathbf{p}| < k+1/2} \mathcal{F}[\mathbf{v}_\alpha](-\mathbf{p}) \cdot \mathcal{F}[(\mathbf{v}_\beta \cdot \nabla) \mathbf{v}_\gamma](\mathbf{p}), \quad (2)$$

and the energy flux  $\Pi_{\alpha\beta\gamma}(k) = - \int_0^k T_{\alpha\beta\gamma}(k) dk$  for  $(\alpha, \beta, \gamma) \in \{c, i\}$ .  $\mathcal{F}[\mathbf{v}](\mathbf{k})$  expresses the Fourier transform of  $\mathbf{v}$ .

Figure 7 in ref.[5] shows the energy fluxes normalized by the dissipation rate  $\Pi(k)/\langle\epsilon\rangle$  versus  $k\eta$ , together with the total flux denoted by  $\Pi_{\text{ttt}}$ . We find that, all along the inertial range, the coherent flux coincides with the total one and the other fluxes are almost zero. In the dissipative range, the coherent flux still dominates, though it begins to depart from the total one, since  $\Pi_{\text{cci}}$  and  $\Pi_{\text{icc}}$  start to build up. The fluxes  $\Pi_{\text{cci}}$  and  $\Pi_{\text{icc}}$  tend to compensate each other with increasing  $k\eta$ . The remaining terms are negligible.

#### 4.6 Velocity flatness

We examine the relationship between the scale dependent flatness of wavelet coefficients for the total velocity field and the scale dependent compression rate defined by the percentage of wavelet coefficients corresponding to the coherent vortices at each scale. Figure 6 in ref.[5] shows that the flatness increases with the wavenumber. The scale dependent compression rate is plotted by the symbol  $\bigcirc$  in fig. 10 in ref.[5]. For larger scales, almost all coefficients are retained by the coherent part, while the rate decreases for this range  $k_j\eta \gtrsim 0.1$ . So, the wavelet representation detects the flow intermittency, which means that the spatial support of active regions decreases with scale.

### 5 Influence of the Reynolds number from $R_\lambda = 167$ to 732

We examine the influence of the Reynolds number on the overall compression rate and the number of the wavelet coefficients corresponding to the coherent vortices.

## 5.1 Compression rate

The overall compression rate is the percentage of the coherent wavelet coefficients which are kept. Figure 8(top) in ref.[5] shows the  $R_\lambda$  dependence of the compression rate. The compression rate decreases monotonically from 3.6% to 2.6% according to  $C \propto R_\lambda^{-0.21}$ . This reflects the fact that the flow intermittency increases with  $R_\lambda$  which is shown in the previous experimental results presented in [9]. The exponent is estimated by a least square fit of the four available data points. Thus, we conjecture that the wavelet representation become the more efficient with increasing  $R_\lambda$ .

## 5.2 Degree of freedom

Figure 8(bottom) in ref.[5] shows the number of retained coefficients for the total and the coherent parts versus  $R_\lambda$ . As the overall compression rate decreases monotonically with increasing  $R_\lambda$ , the number of coefficients of the coherent part grows slower than that of the total flow obtained by DNS.

## 6 Conclusion

We have applied the CVE method to DNS data of homogeneous isotropic turbulence for different Taylor microscale Reynolds numbers, ranging from  $R_\lambda = 167$  to 732, in order to study the role of coherent and incoherent vorticity fields with respect to the flow intermittency. We have shown that few wavelet coefficients are sufficient to represent the coherent vortices which preserve the total flow in the inertial range, while the large majority of the coefficients corresponds to an incoherent background flow, which is structureless. We find that, as the Reynolds number increases, the percentage of wavelet coefficients representing the coherent vortices decreases. Although the number of degrees of freedom necessary to track the coherent vortices remains small, they preserve the nonlinear dynamics of the flow. Thus it is conjectured that the wavelet representation could reduce the number of degrees of freedom to compute fully developed turbulent flows in comparison to the standard estimation based on Kolmogorov's theory.

The present results motivate further developments of the Coherent Vortex Simulation (CVS)[10]. The present estimation shows that CVS

might become more efficient as the Reynolds number increases, since the percentage of retained coherent modes decreases. First results of CVS for a three-dimensional turbulent mixing layer are shown in ref.[10].

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## 参考文献

- [1] M. Farge, G. Pellegrino and K. Schneider, “Coherent vortex extraction in 3d turbulent flows using orthogonal wavelets,” *Phys. Rev. Lett.*, **87** 45011 (2001).
- [2] M. Farge, K. Schneider, G. Pellegrino, A. Wray and B. Rogallo, “Coherent vortex extraction in three-dimensional homogeneous turbulence: comparison between CVS-wavelet and POD-Fourier decompositions,” *Phys. Fluids*, **15** 2886 (2003).
- [3] M. Farge and K. Schneider, “Coherent vortex simulation (CVS), a semi-deterministic turbulence model using wavelets,” *Flow, Turbulence and Combustion*, **66**, 393 (2001).
- [4] M. Farge, K. Schneider and N. Kevlahan, “Non-Gaussianity and coherent vortex simulation for two-dimensional turbulence using an adaptive orthonormal wavelet basis,” *Phys. Fluids*, **11** 2187 (1999).
- [5] Okamoto, N, Yoshimatsu, K, Schneider, K, Farge, M, and Kaneda, Y (2007) Coherent vortices in high resolution direct numerical simulation of homogeneous isotropic turbulence : A wavelet viewpoint, *Physics of Fluids* **19**, 115109.
- [6] D. Donoho and I. Johnstone, “Ideal spatial adaptation via wavelet shrinkage,” *Biometrika* **81**, 425 (1994).



- [7] M. Yokokawa, K. Itakura, A. Uno, T. Ishihara, and Y. Kaneda, “16.4-Tflops direct numerical simulation of turbulence by a Fourier spectral method on the Earth Simulator,” in *Proc. IEEE/ACM SC2002 Conf. (CD-ROM)*, Baltimore, 2002; <http://www.sc-2002.org/paperpdfs/pap.pap273.pdf> (2002).
- [8] Y. Kaneda, T. Ishihara, M. Yokokawa, K. Itakura and A. Uno, “Energy dissipation rate and energy spectrum in high resolution direct numerical simulations of turbulence in a periodic box,” *Phys. Fluids* **15**, L21 (2003).
- [9] K. R. Sreenivasan and R. A. Antonia, “The phenomenology of small-scale turbulence,” *Annu. Rev. Fluid Mech.*, **29** 435 (1997).
- [10] K. Schneider, M. Farge, G. Pellegrino, and M. Rogers, “Coherent vortex simulation of 3d turbulent mixing layers using orthogonal wavelets,” *J. Fluid Mech.*, **534** 39 (2005).